Enrollment No: ______________________ **Exam Seat No:**______________________

C.U.SHAH UNIVERSITY

Winter Examination-2022

Subject Name: Problem Solving -I

Subject Code: 5SC02PRS1 **Branch: M.Sc. (Mathematics)**

Semester: 2 **Date: 21/09/2022** **Time: 11:00 To 02:00** **Marks: 70**

Instructions:

(1) Use of a Programmable calculator and any other electronic instrument is prohibited.
(2) Instructions written on the main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

SECTION – I

Q-1 **Attempt the Following questions** **[07]**

a. True/False: Any subset of linearly independent setis linearly. independent set. (01)

b. Check whether the set $\{(4,5,3), (1,0,2), (0,0,0)\}$ is basis of $R^3(R)$? (02)

c. Find C.F. of $(D^2 - 4D + 4)y = x^3 e^{2x}$. (02)

d. Check whether the set $\{(x, y, z) \in R^3 : x + y = 1\}$ is a subspace of R^3 or not? (02)

Q-2 **Attempt all questions** **[14]**

a. Let W_1, W_2 and W_3 be subspaces of a vector space V such that $W_2 \subseteq W_1$, show that $W_1 \cap (W_2 + W_3) = W_2 + (W_1 \cap W_3)$. (05)

b. Let $V = \{f \mid f: R \to R\}$ be a vector space over R . V_e and V_o be the set of even and odd functions respectively. Show that V_e and V_o are subspaces of V and $V = V_e \oplus V_o$. (05)

c. Find modulus and argument of the complex number $z = (4 + 2i)(-3 + \sqrt{2}i)$. (04)

OR

Q-2 **Attempt all questions** **[14]**

a. Find complex number z if arg $(z + 1) = \frac{\pi}{6}$ and $\arg(z - 1) = \frac{2\pi}{3}$. (05)

b. Check whether the function $f(z) = \begin{cases} \frac{Re\ (z)}{z} & z \neq 0 \\ 0 & z = 0 \end{cases}$ is continues at 0 or not ? (05)

c. Find $P.I.$ of $(D^2 + 5D + 4)y = x^2 + 9x + 4$. (04)

Q-3 **Attempt all questions.** **[14]**

a. Solve: $(D^4 + 2D^2 - 3D)y = 3\,e^{2x} + 4 \sin x$. (06)

b. Solve: $(2xy + y - \tan y)\,dx + (x^2 - x\tan^2 y + \sec^2 y)dy = 0$. (06)

c. Solve : $\frac{d^3y}{dx^3} - 4\frac{d^2y}{dx^2} + 5\frac{dy}{dx} - 2y = 0$. (02)

OR

Q-3 **[14]**

a. Find a matrix P that diagonalize $A = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$. Hence find A^{13}. (07)

b. Find the rank of matrix by normal form, where $A = \begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$. (04)

c. Check whether the set $\{1, i\}$ is linearly dependent in vector space $C(C)$or not ? (03)

SECTION – II

Q-4 **Attempt the Following questions** **[07]**

a. Let V denotes the vector space of 7×7 real skew symmetric matrices then $\dim V =$ ___. (01)

b. Solve $(xy^2 + x)dx + (yx^2 + y)dy = 0$. (02)

c. Find the two numbers whose sum is 4 and product is 8. (02)

d. Let A be an 3×3 matrix with eigenvalues 1, -1,0. Then the determinant of $I + A^{100}$ is____ . (02)

Q-5 **Attempt all questions** **[14]**

a. For which values of $'a'$will the following system have no solutions? Exactly one solution ?Infinitely many solutions ? (06)

$x + 2y - 3z = 4\,, 3x - y + 5z = 2\,, 4x + y + (a^2 - 14)z = a + 2$.

b. Let T be a linear transformation on R^2 defined by $T(x, y) = (2x + y, 3x + 2y)$.Show that T is invertible and find T^{-1}. (06)

c. Evaluate : $\int_{|z+1|=2} \frac{z^2}{4-z^2}dz$. (02)

OR

Q-5 **Attempt all questions** **[14]**

a. Solve: $x^4\frac{d^3y}{dx^3} + 2x^3\frac{d^2y}{dx^2} - x^2\frac{dy}{dx} + xy = 1$. (06)

b. Solve : $x^2y\,dx - (x^3 + y^3)\,dy = 0$. (05)

c. Evaluate: $\int_C \frac{\tan\left(\frac{z}{2}\right)}{(z-1-i)^2}dz$; C: Rectangle with vertices at $\pm(2 + 2i)$. (03)

Q-6 **Attempt all questions** **[14]**

a. Let A be 6×6 matrix over R with characteristic polynomial $(x-3)^2(x-2)^2$ and minimal polynomial $(x-3)(x-2)^2$.Find Jordan Canonical form of A. (06)

b. Show that $f(z) = |z|^2$ is differentiable only at origin. (05)

c. Express the number $z = \frac{1}{2-3i} + \frac{5-i}{6+2i}$ into $x + iy$ form. (03)

OR

Q-6 **Attempt all Questions** **[14]**

a. Show that $u(x,y) = e^{-2xy}\sin(x^2 - y^2)$ is a harmonic function. Find the conjugate function $v(x,y)$ and an analytic function $f(z)$ for which $u(x,y)$is the real part. (07)

b. If $f(z) = \frac{z}{(z-2)(z+i)}$ then expand $f(z)$ in Laurent series in powers of z in the region $1 < |z| < 2$. (07)